Understanding Assessment & Improving Delivery International A Level Mathematics

YMA01-25IO2/02

Your Trainer Today is: Joe Skrakowski

Welcome to this Professional Development Training

- Designed for teachers teaching, or who are looking to teach the Pearson Edexcel International A Level specification
- To introduce you to the idea of assessment objectives, what are they and why they are used when writing examination papers
- Analyse recent question papers and learn which types of question match the different assessment objectives
- Investigate different assessment objectives, considering how questions in these areas have been answered by looking at feedback from previous exam series
- Discuss strategies for teaching to try and make sure students can access questions targeting different assessment objectives,
- Look at how we mark candidates' work
- Review the support Pearson offers for the qualification,

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Introduction to assessment

Problems and examples in this presentation

Edexcel exam questions undergo a rigorous process before any student sees the examination paper.

In several slides in this presentation the language and style are not fully that of the exams – indeed there are some problems that would not do at all as exam questions but do have a use as a teaching application.

The questions themselves are indicative also of the range that students should see in class.

They are not intended in any way as a ‘pointer’ to examination questions.
The Edexcel team have produced material which teachers will be able to use to support their teaching – especially of the new topics.

General Structure of an Assessment

Content	Assessment Objectives
• Facts	Demonstrate knowledge of facts, techniques and relationships
• Techniques	Demonstrate application of facts, techniques and relationships to solve problems
• Relationships	Demonstrate processes to model real situations and to interpret results of calculations involving models
• Models	

General Structure of an Assessment

Content coverage

- sufficient for each separate assessment (samples from (nearly) all sections of the content list)
- complete coverage over a cycle of assessments

Assessment Objectives

- fixed from assessment to assessment
- same weightings from assessment to assessment (some leeway allowed)

Structure of the Edexcel P1 Assessment

- Content - as given in the specification
- e.g.. Laws of indices for all rational exponents.
- e.g.. Interpret linear and quadratic inequalities graphically.
- e.g.. Solve simultaneous equations; analytical solution by substitution.

Assessment Objectives (AOs)

1. Recall, select and use their knowledge of mathematical facts, concepts and techniques..... DO [AO1]

2. Construct rigorous mathematical arguments and proofs... PROVE [AO2]

3. Recall, select and use their knowledge of standard mathematical models to represent situations in the real world.... MODEL [AO3]

4. Comprehend translations of common realistic contexts into mathematics..... INTERPRET results [AO4]

5. Use contemporary calculator technology and other permitted resources........ CALCULATE / FIND [AO5]

Structure of the Edexcel pure units assessment

All figures in the following table are expressed as marks out of 75.

	AO1	AO2	AO3	AO4	AO5
P1	30–35	25–30	5–15	5–10	1–5
P2	30–35	25–30	5–15	5–10	1–5
P3	30–35	25–30	5–15	5–10	1–5
P4	30–35	25–30	5–15	5–10	1–5

Structure of the Edexcel applied units assessment

All figures in the following table are expressed as marks out of 75.

	AO1	AO2	AO3	AO4	AO5
M1	20 – 25	20 – 25	15 – 20	6 – 11	4 – 9
M2	20 – 25	20 – 25	10 – 15	7 – 10	5 –10
S1	20 – 25	20 – 25	15 – 20	5 – 10	5 –10
S2	20 – 25	20 – 25	10 – 15	5 – 10	5 –10
D1	20 – 25	20 – 25	15 – 20	5 – 10	5 –10

Structure of the Edexcel P1 assessment

- In practice most questions
- on our examination papers have more than one AO assigned to them.

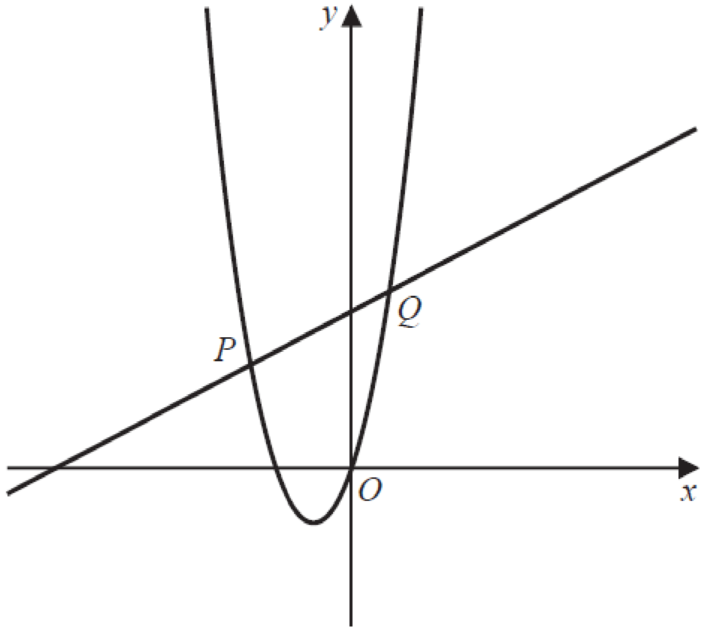


Figure 1

Figure 1 shows a sketch of the curve with equation $y = 2x^2 + 3x$ and the straight line with equation $y = \frac{1}{2}x + 3$

The line meets the curve at the points P and Q, as shown in Figure 1.

(a) Using algebra, find the coordinates of P and the coordinates of Q.

(5)

(b) Hence write down the range of values of x for which $2x^2 + 3x \geqslant \frac{1}{2}x + 3$

Activity 1

Initial assignment of AOs to two questions.
Work through each question

The first Q has the associated mark scheme.
Use it to assign the marks to AOs

Then do the second Q

Structure of the Edexcel P1 assessment

(a)

- $2x^2 + 3x = \frac{1}{2}x + 3$ AO1 (1 mark)
- $4x^2 + 5x - 6 = 0$ AO2 (2 marks)
- $(4x - 3)(x + 2) = 0$ AO1 (1 mark)

- $x = \frac{3}{4}$ or $x = -2$

- $y = \frac{1}{2} \times \frac{3}{4} + 3 = 29/8$ or $y = \frac{1}{2} \times (-2) + 3 = 2$ AO1 (1 mark)

(b)

$x \leq \frac{3}{4}$ or $x \geq -2$ AO4 (2 marks)

Structure of the Edexcel P1 assessment

- Decide how to allocate the marks to the AOs
- Which elements of the content does the question require?

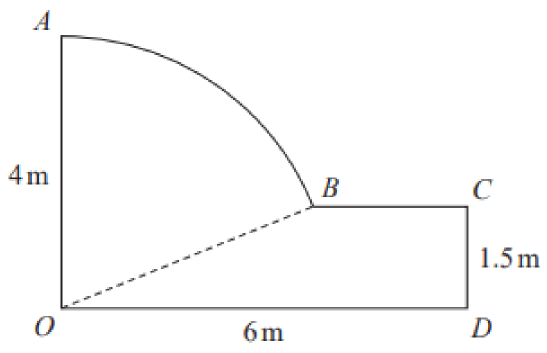


Figure 1

Figure 1 shows the plan for a garden.

In the plan

- OA and CD are perpendicular to OD
- AB is an arc of the circle with centre O and radius 4 metres
- BC is parallel to OD
- OD is 6 metres, OA is 4 metres and CD is 1.5 metres

(a) Show that angle AOB is 1.186 radians to 4 significant figures. **(2)**

AO1 [1]AO2 [1]

(b) Find the perimeter of the garden, giving your answer in metres to 3 significant figures. **(4)**

AO4 [1] AO1 [2] AO3 [1]

(c) Find the area of the garden, giving your answer in square metres to 3 significant figures. **(4)**

AO4 [1] AO1 [1] AO3 [2]

AO1

Looking at AO1 on P1, P2, P3 and P4

Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts

Looking at AO1 on P1, P2, P3 and P4

Knowledge / Recall

Simplify $a \times (\sqrt{a})^{-1}$

Write down an equation of the straight line with gradient 4 passing through P (2, 1).

Knowledge / Recall

Find the area of a sector radius 3cm and angle at the centre 0.4 radians

Looking at AO1 on P1, P2, P3 and P4

Concepts

If $y = x^3$ then $y' = 3x^2$ so $\int x^2\, dx = \frac{x^3}{3} +$ C

Concepts

If $ab = 0$ then $a = 0$ or $b = 0$

If $\log_a b = n$ then $a^n = b$

Looking at AO1 on P1, P2, P3 and P4

Techniques

Given $y = 48x - 10x^2$ find the maximum value of y

Techniques

Write $\dfrac{7}{(x-3)(2x+1)}$ as a sum of partial fractions

Looking at AO1 on P1, P2, P3 and P4

Activity 2

Use the sheet for activity 2 to enter some ideas of:

- Knowledge/recall
- Concepts
- Techniques
- from the specification for Algebra and Functions from Module P1.

Looking at AO1 on P1, P2, P3 and P4

Looking at AO1 on P1, P2, P3 and P4

Questions which only assess AO1 are rare:

- e.g.. Specimen Pure 1 Q1 (Differentiation, integration)
- e.g.. Practice Pure 1 Q1 (Transformations)
- e.g.. Specimen Pure 2 Q1a (Binomial expansion)
- e.g.. June 23 Pure 1 Q1 (Differentiation)
- e.g.. June 23 (P4) Q1(a) (Binomial Expansion)

Looking at AO1 on P1, P2, P3 and P4

An example

3. A curve C has parametric equations

$$x = \sqrt{3}\tan\theta, \quad y = \sec^2\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{3}$$

The cartesian equation of C is

$$y = f(x), \quad 0 \leqslant x \leqslant k, \quad \text{where } k \text{ is a constant}$$

(a) State the value of k.

(1)

(b) Find $f(x)$ in its simplest form.

(2)

(c) Hence, or otherwise, find the gradient of the curve at the point where $\theta = \frac{\pi}{6}$

(3)

Looking at AO1 on P1, P2, P3 and P4

Activity 3

- Look briefly through Activity 3
- The 3 exam questions were given AO1 only
- Decide on the Knowledge/Concepts/Techniques being assessed
- Do you agree with the assignment of only AO1?

Looking at AO1 on P1, P2, P3 and P4

However AO1 appears in most questions:

-as an explicit part (a)

- as underlying knowledge/skills/ concepts.

2. (a) Find $\int \frac{4x+3}{x} \, dx, \quad x > 0$

AO1

(2)

(b) Given that $y = 25$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(4x+3)y^{\frac{1}{2}}}{x} \qquad x > 0, y > 0$$

giving your answer in the form $y = [g(x)]^2$.

AO2

(5)

AO2

Looking at AO2 on P1, P2, P3 and P4

Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.

Looking at AO2 on P1, P2, P3 and P4

Arguments and Proofs

Construct rigorous arguments and proofs'

- An argument is a series of statements which support a belief/hypothesis.
- Arguments can be **deductive** – using the laws of logical reasoning
- Or **inductive** – using evidence/observation to support the hypothesis.
- (mathematical) proofs are discussed in subsequent slides .

Looking at AO2 on P1, P2, P3 and P4

A proof must show all **assumptions** you are using, have a clear **sequential list of steps** that logically follow, and must cover **all possible cases**.
You should usually make a **concluding statement**, e.g.. restating the original conjecture that you have proven.

a. Proof by Deduction

This is the simplest type, where you start from known facts and reach the desired conclusion via deductive steps.

"Prove that the product of two odd numbers is odd."
Let p, q be integers, then $2p + 1$ and $2q + 1$ are odd numbers.

$$(2p + 1)(2q + 1) = 4pq + 2p + 2q + 1$$
$$= 2(2pq + p + q) + 1$$

This is one more than a multiple of 2, and is therefore odd.

Looking at AO2 on P1, P2, P3 and P4

Proof by Deduction requires you to start from known facts and end up at the conclusion. It is not acceptable to start with to the conclusion, and verify it works, because you are assuming the thing you are trying to prove.

Example: Prove that if three consecutive integers are the sides of a right-angled triangle, they must be 3, 4 and 5.

Incorrect Proof:

1

2

3

5

Correct Proof:

4

Looking at AO2 on P1, P2, P3 and P4

Types of Proof

a. Proof by Deduction

Exam Tip: This is quite a common last part.

Prove that $x^2 + 4x + 5$ is positive for all values of x.

$$x^2 + 4x + 5 = (x+2)^2 + 1$$

$(x+2)^2 \geq 0$ for all x

$$\therefore (x+2)^2 + 1 \geq 1 > 0$$

Anything squared is at least 0. This is formally known as the '*trivial inequality*'.

Test Your Understanding

Prove that the sum of the squares of two consecutive odd numbers is 2 more than a multiple of 8.

Let $2n-1$ and $2n+1$ be any two consecutive odd numbers, where n is an integer.

$$(2n-1)^2 + (2n+1)^2 = 4n^2 - 4n + 1 + 4n^2 + 4n + 1$$

$= 8n^2 + 2$ which is 2 more than a multiple of 8.

Looking at AO2 on P1, P2, P3 and P4

(a) Show that (x – 2) is a factor of $x^3 - x^2 - x - 2$

(b) Show that the equation $x^3 = x^2 + x + 2$ has exactly one real root

‘Show’ could be replaced by ‘Prove’.

Looking at AO2 on P1, P2, P3 and P4

(a) Let $\mathrm{P}(x) = x^3 - x^2 - x - 2$

P(2) = 8 – 4 – 2 – 2 = 0

So, by the factor theorem (x – 2) is a factor of P(x)

Or using polynomial division (no remainder)

$$x - 2\,\overline{\big)\,x^3 - x^2 - x - 2}\quad \text{quotient: } x^2 + x + 1$$

Looking at AO2 on P1, P2, P3 and P4

(b) $x^3 = x^2 + x + 2$

$$\Rightarrow x^3 - x^2 - x - 2 = 0$$

$$\Rightarrow (x-2)(x^2+x+1) = 0$$

$$\Rightarrow x = 2 \text{ or } x^2 + x + 1 = 0$$

The quadratic has no real roots as the discriminant is – 3

Hence the equation $x^3 = x^2 + x + 2$ has only one real root.

There **MUST** be a conclusion to complete.

Looking at AO2 on P1, P2, P3 and P4

b. Proof by Exhaustion

This means breaking down the statement into all possible smaller cases, where we prove each individual case.
(This technique is sometimes known as 'case analysis')

Prove that $n^2 + n$ is even for all integers n.

Looking at AO2 on P1, P2, P3 and P4

Proof (exhaustion) Example

Prove that any square number when divided by 5 leaves a remainder of 0 or 1 or 4

Looking at AO2 on P1, P2, P3 and P4

Proof (exhaustion) Exhaustive sets

Problems which involve multiplication or division properties by a particular number can often be tackled by using an algebraic exhaustive set.

For example:

Show that the square of any whole number is a multiple of 3 or one more than a multiple of 3

Looking at AO2 on P1, P2, P3 and P4

Proof (exhaustion) Example

SAMs (IAL – P2)
Prove, by exhaustion, that $n^2 + 2$ is not divisible by 4.
(4 marks)

Looking at AO2 on P1, P2, P3 and P4

SAMs Solution

n	n^2	n^2+2	
1	**1**	**3**	**Odd**
2	**4**	**6**	**Even**
3	**9**	**11**	**Odd**
4	**16**	**18**	**Even**
5	**25**	**27**	**Odd**
6	**36**	**38**	**Even**

When n is odd, n^2 is odd (odd $\times$ odd $=$ odd) so n^2+2 is also odd	M1
So for all odd numbers n, n^2+2 is also odd and so cannot be divisible by 4 (as all numbers in the 4 times table are even)	A1
When n is even, n^2 is even **and a multiple** of 4, so n^2+2 cannot be a multiple of 4	M1
Fully correct and exhaustive proof. Award for both of the cases above plus a final statement "So for all n, n^2+2 cannot be divisible by 4"	A1*
	(4)

Looking at AO2 on P1, P2, P3 and P4

c. Disproof by Counter-Example

While to prove a statement is true, we need to prove every possible case (potentially infinitely many!), we only need one example to disprove a statement.
This is known as a counterexample.

Disprove the statement:
"$n^2 - n + 41$ is prime for all integers n."

If $n = 41$, then we have $41^2 - 41 + 41$
$$= 41^2$$
Which is not prime as it has a factor of 41.
Thus the statement is not true.

Looking at AO2 on P1, P2, P3 and P4

A common use of deductive proof is in that of trigonometric identities.

(a) Prove that

$$\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x, \qquad x \neq (2n+1)90°, n \in \mathbb{Z}$$

(3)

	Examples:	
(a)	$\frac{1-\cos 2x}{1+\cos 2x} = \frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)} = \frac{2\sin^2 x}{2\cos^2 x} = \tan^2 x$	M1dM1A1

This solution shows every step so is fully acceptable as a proof

Looking at AO2 on P1, P2, P3 and P4

(a) Prove that

$$\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x, \qquad x \neq (2n+1)90°, n \in \mathbb{Z}$$

(3)

$$\frac{1-\cos 2x}{1+\cos 2x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

M0dM0A0

$$\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x \Rightarrow 1-\cos 2x = \tan^2 x(1+\cos 2x)$$

$$1-(1-2\sin^2 x) = \tan^2 x(1+2\cos^2 x-1)$$

$$2\sin^2 x = \frac{\sin^2 x}{\cos^2 x}(2\cos^2 x)$$

$$2\sin^2 x = 2\sin^2 x$$

M1dM1A0

How many marks should these approaches get?

Looking at AO2 on P1, P2, P3 and P4

Proof (by deduction) Trig identities

Ideally:

- Start with the more complex side of the identity.
- Simplify the more complex to the less complex side.

There should be a conclusion to complete in this case

Looking at AO2 on P1, P2, P3 and P4

Proof (by deduction) Trig identities

Proof by deduction requires you to start from **known facts**

What are these?

Knowledge of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, and $\sin^2\theta + \cos^2\theta = 1$. In the Specification for P2

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2\sin A\cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2\tan A}{1 - \tan^2 A}$

As required knowledge for P3

Looking at AO2 on P1, P2, P3 and P4

Proof (by deduction) Trig identities

Activity 5

Log and Trig proofs

Activity 5 has examples of attempted proofs of two fairly standard results – one from P2 and one from P3

Look through the proofs and decide whether they are valid or not

Looking at AO2 on P1, P2, P3 and P4

Proof By Contradiction

! To prove a statement is true by contradiction:

- **Assume** that the statement is in fact **false**.
- Prove that this would **lead to a contradiction**.
- Therefore we were wrong in assuming the statement was false, and therefore it must be true.

Prove that there is no greatest odd integer.

? Assumption

? Show contradiction

? Conclusion

How to structure/word proof:

1. "Assume that *[negation of statement]*."
2. [*Reasoning followed by…*] "*This contradicts the assumption that…*" or "*This is a contradiction*".
3. "Therefore [*restate original statement*]."

Looking at AO2 on P1, P2, P3 and P4

Negating the original statement

The first part of a proof by contradiction requires you to negate the original statement. What is the negation of each of these statements?

"There are infinitely many prime numbers."

"All teachers are clever"

"If it is raining, my garden is wet."

"There are infinitely many prime numbers."	"All teachers are clever"	"If it is raining, my garden is wet."
"There are infinitely many non-prime (i.e. composite) numbers."	"There exists a teacher who is not clever."	"If it is not raining, my garden is dry."
"There are finitely many prime numbers."	"No teachers are clever."	"If it is not raining, my garden is wet."
"There are finitely many non-composite numbers."	"The trainer is clever."	"If it is raining, my garden is not wet."

Comments: The negation of "all are" is not "none are". So the negation of "everyone likes green" wouldn't be "no one likes green", but: "not everyone likes green". Do not confuse a 'negation' with the 'opposite'.

Comments: If you have a conditional statement like "*If A then B*", then the negation is "*If A then not B*", i.e. the same condition applies, but the implication is negated.

Looking at AO2 on P1, P2, P3 and P4

More Examples

Prove by contradiction that if n^2 is even, then n must be even.

? Assumption

? Show contradiction

? Conclusion

Looking at AO2 on P1, P2, P3 and P4

Proof on P4

- Students should be familiar with the proofs of the infinity of prime numbers and with the irrationality of the square root of 2 In the Spec
- It's then easy to prove, for example that $(\sqrt{2} + 1)$ is irrational – by contradiction
- Students don't find it easy to adapt the proof of the irrationality of $\sqrt{2}$ to, for example, $\sqrt{3}$ – but should be encouraged to do so.
- Students should also be encouraged to decide why the 'proof' breaks down for $\sqrt{4}$.

Looking at AO2 on P1, P2, P3 and P4

How to structure proof by contradiction

First stages

Development

Final stages

Looking at AO2 on P1, P2, P3 and P4

Proof ' in the units – a summary

- P1 – no formal proofs, but 'show that's'' may be set
- P2 – 1.1 (deductive proof),1.2 (proof by exhaustion) and 1.3 (Disproof by counter example)
- P3 – 2.2 and 2.3 (proofs of trigonometric identities)
- P4 – 1.1 (proof by contradiction)

Looking at AO2 on P1, P2, P3 and P4

Looking at AO2 on P1, P2, P3 and P4

'Extended arguments involving the manipulation of mathematical expressions.'

Stage 1 — Translation and understanding

⇩

Stage 2 — Calling upon relevant knowledge

⇩

Stage 3 — Planning a strategy

⇩

Stage 4 — Execution and interpretation

Looking at AO2 on P1, P2, P3 and P4

‘Extended arguments involving the manipulation of mathematical expressions.’

…..whether the answer is given or not:
‘Show that…’ instead of
‘Find’

We could claim that ‘Show that’ is easier than ‘Find’ because it gives the student a definite end point

We could claim that ‘Show that’ is harder than ‘Find’ because it could force the student to use a specific method.

For an argument supporting reasons do not usually have to be given

‘Find’ is much more common – for example in the SAMS there are 49 of them (13 ‘Show that’s’ and 5 ’Proves’)

Looking at AO2 on P1, P2, P3 and P4

Scaffolding

Scaffolding is the term used to add structure to a question which will usually require extended mathematics.

$$f(x) = \frac{6 - 5x - 4x^2}{(2 - x)(1 + 2x)} \qquad x > 2$$

Prove that $f(x)$ is a decreasing function.

Just think for a moment about what strategies would students plan to use to answer this question……

The question was eventually scaffolded as………

Looking at AO2 on P1, P2, P3 and P4

Scaffolding

Scaffolding is the term used to add structure to a question which will usually require extended mathematics to answer it.

Gives a start. Makes it clear what is required

$$\frac{6 - 5x - 4x^2}{(2 - x)(1 + 2x)} \equiv A + \frac{B}{(2 - x)} + \frac{C}{(1 + 2x)}$$

(a) Find the values of the constants A, B and C.

$$f(x) = \frac{6 - 5x - 4x^2}{(2 - x)(1 + 2x)} \qquad x > 2$$

(b) Using part (a), find $f'(x)$.

(c) Prove that $f(x)$ is a decreasing function.

Looking at AO2 on P1, P2, P3 and P4

Scaffolding

As well as making problems more accessible, scaffolding allows specific mathematical techniques to be examined.

A curve has equation

$$y = \ln(1 - \cos 2x), \qquad x \in \mathbb{R},\ 0 < x < \pi$$

find the exact coordinates of the point on the curve where $\dfrac{dy}{dx} = 2\sqrt{3}$

No scaffolding.
This Q could be classed as basically all AO2

Looking at AO2 on P1, P2, P3 and P4

As well as making problems more accessible, scaffolding allows specific mathematical techniques to be examined.

A curve has equation

$$y = \ln(1 - \cos 2x), \qquad x \in \mathbb{R},\ 0 < x < \pi$$

Show that

(*a*) $\dfrac{dy}{dx} = k \cot x$, where k is a constant to be found.

Tests differentiation and a trig identity

Hence find the exact coordinates of the point on the curve where

(*b*) $\dfrac{dy}{dx} = 2\sqrt{3}$

Tests solution of a trig equation.

AO3

Looking at AO3 on P1, P2, P3 and P4

Recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models.

Looking at AO3 on P1, P2, P3 and P4

AO3 refers to the use of standard models – this means models that are commonly known or models that are straightforward to understand.

Here is an example from the material in P1

At 12:00 a ship is at a point A 48 km West of a port P.

The ship sails on a bearing of 060° to a point B.

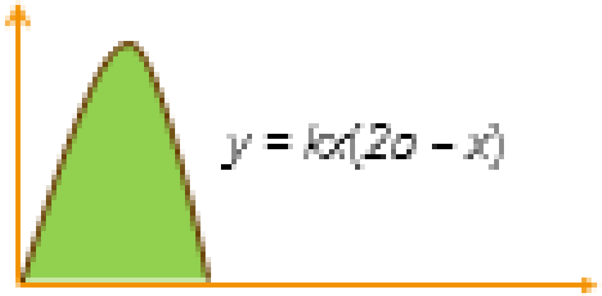
PB = 36 km

Find by calculation, the two possible bearings of B from P.
Give your answers correct to the nearest degree.

Looking at AO3 on P1, P2, P3 and P4

AO3 refers to the use of standard models – this means models that are commonly known or models that are straightforward to comprehend.

Here is an example from the material in P1



The diagram represents the cross-section of a tunnel.

The width of the cross section is 10 m and the height is 6 m

(a) Find the value of k and the value of a.

(b) Use integration to find the area of the cross-section of the tunnel.

Looking at AO2 on P1, P2, P3 and P4

AO3 refers to the use of standard models – this means models that are commonly known or models that are straightforward to comprehend.

Common models on P2 require the use of Arithmetic or Geometric series for describing growth and decay.

Looking at AO3 on P1, P2, P3 and P4

Here is an typical example from the material in P3

A rare species of mammal is being studied. The population P, t years after the study started, is modelled by the formula

$$P = \frac{900e^{\frac{1}{4}t}}{3e^{\frac{1}{4}t} - 1}, \qquad t \in \mathbb{R}, \qquad t \geqslant 0$$

Looking at AO3 on P1, P2, P3 and P4

Here is an example from the material in P4

Students need to understand modelling using differential equations.
The key idea is that $\frac{\mathrm{d}A}{\mathrm{d}t}$ represents the rate of change of the quantity A

Often the form of the rate of change is given

Students do need to be able to write down/derive or interpret a D.E. which shows the rate to change of A to be:

- constant (with interpretation of a negative sign)
- to be proportional to A.

Overlaps with AO4

AO4

Looking at AO4 on P1, P2, P3 and P4

Comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.

Overlaps with AO3

'Hence' is often a key word in AO4

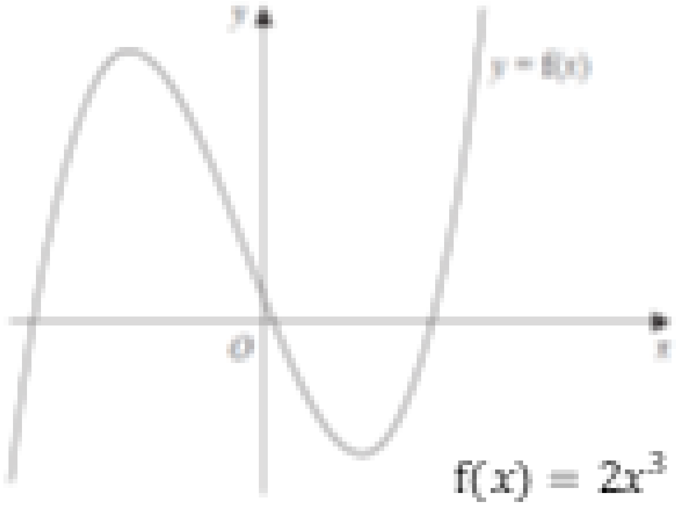
'Deduce' is often a key word in AO4

Looking at AO4 on P1, P2, P3 and P4

Here is an example from the material in P1

....use the results of calculations to make predictions......

4.



$$f(x) = 2x^3 + \frac{3}{2}x^2 - 18x + 3$$

(a) Find the set of values of x for which $f(x)$ is decreasing

(b) Hence find the number of roots of the equation $f(x) = k$ (k constant) according to the values of k

Looking at AO4 on P1, P2, P3 and P4

Here is an example from the material in P2

'....read critically'

Solve the equation $2\log_3(2y+1) - \log_3(2-y) = 1$

explaining clearly why there is only one real solution

Looking at AO4 on P1, P2, P3 and P4

Here is an example from the material in P3

....use the results of calculations to make predictions......

(a) Prove that $\tan x + \cot x \equiv 2\operatorname{cosec} 2x$ for $x \neq n\pi/2$

(b) Deduce that the equation $\tan x + \cot x = 1$ has no real solutions.

Looking at AO4 on P1, P2, P3 and P4

Here is an example from the material in P4

'...use the results of calculations to make predictions.'

A curve has equation

$$y = \ln(1 - \cos 2x), \qquad x \in \mathbb{R}, \ 0 < x < \pi$$

Show that

(*a*) $\frac{dy}{dx} = k \cot x$, where k is a constant to be found.

(4)

Hence find the exact coordinates of the point on the curve where

(*b*) $\frac{dy}{dx} = 2\sqrt{3}$

(4)

AO5

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology...... accurately and efficiently

From the P1 specification

Solution of quadratic equations using the formula
Mensuration and radian measure

Graphs:
- Quadratic
- k/x and k/x^2
- sin, cos and tan

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

From the P2 specification

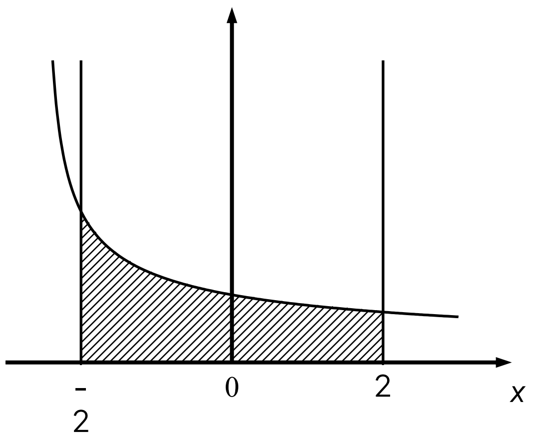
- solution of quadratic inequalities (sum of an Arithmetic Series)
- solution of inequalities requiring taking logs (sum of a Geometric Series)
- values of binomial coefficients
- solution of trig equations – degrees and radians
- evaluation of expressions after integration
- trapezium rule*

*The derivation of the formula is not required knowledge – but should – at least informally – be shown

Looking at AO5 on P1, P2, P3 and P4

The figure shows a sketch of part of the curve C, with equation

The finite region R shown shaded is bounded by C, the *x*-axis and the lines $x = \pm 2$



x	-2	-1	0	1	2
	1		0.4472		0.3333

(a) Complete the table.

(b) Use the trapezium rule to find an estimate of the area of R.

(c) Given that the exact area of R is 2, work out an estimate of the error.

In the exam the language is much more precise!

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

From the P3 specification

Solution of trig equations

Straight line graphs derived from data of the form $y = ax^n$ or $y = kb^x$

Behaviour of f(t) when t gets large. (exponential type models)

*Location of roots of f(x) = 0 by looking for sign changes

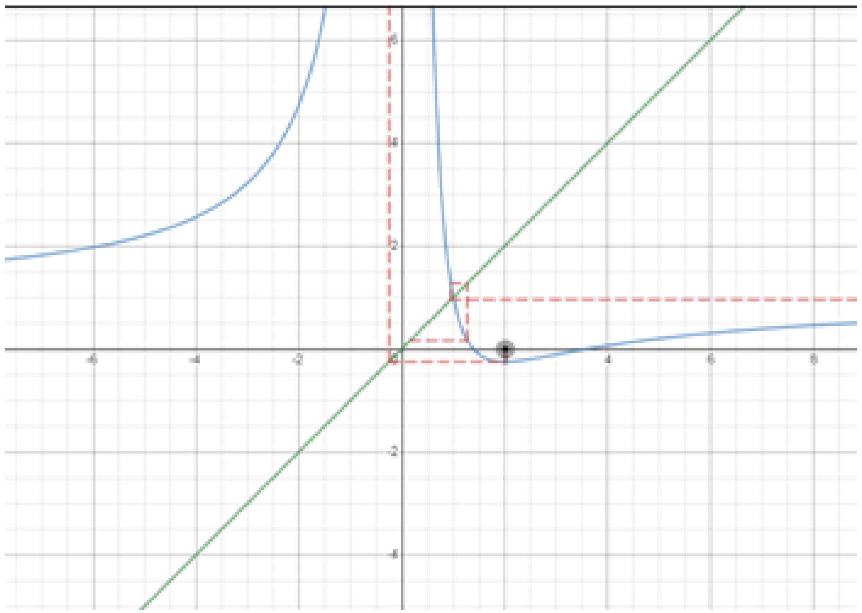
*Iteration

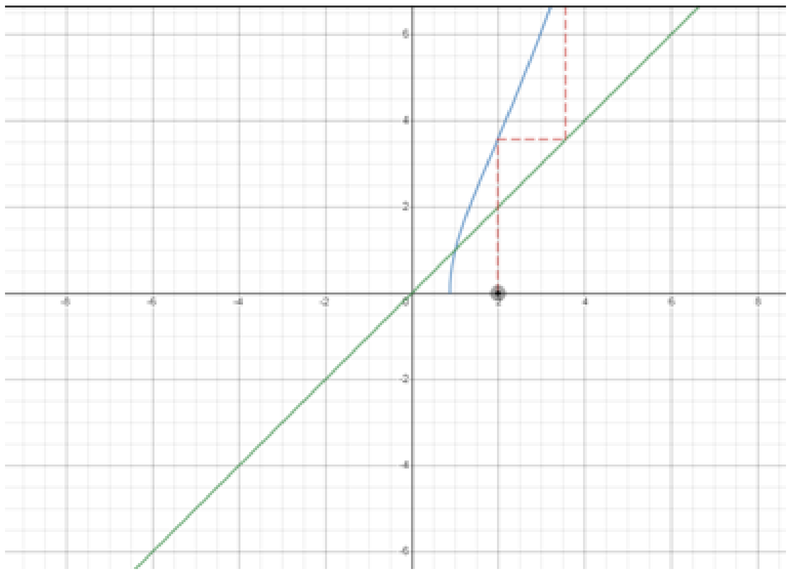
Looking at AO5 on P1, P2, P3 and P4

Iteration

Use contemporary calculator technology…… accurately and efficiently

Students will be given the formula to use
Students should be aware that not all rearrangements of an equation lead to a convergent sequence.





Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

From the P4 specification

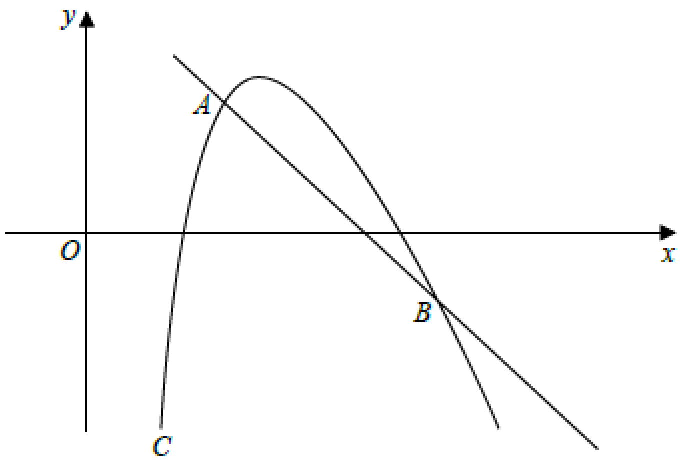
Nothing explicit - finding angles between vectors

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

Consequences:

Specific wording in questions **to prevent** the use of calculators



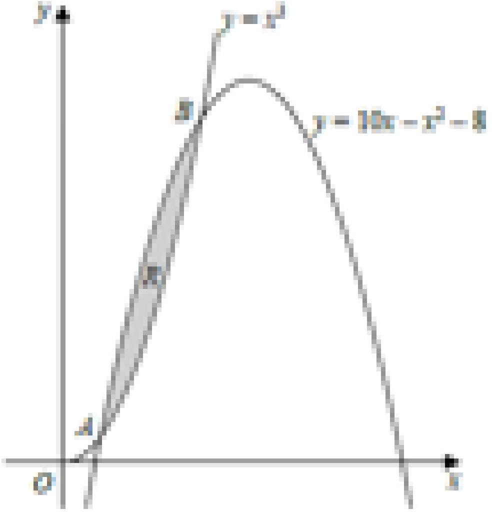
(b) Use algebra to find the coordinates of B.

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

Consequences:

Specific wording in questions to prevent the use of calculators



(b) Use algebra to find the coordinates of the point B

(c) Use calculus to find the exact area of R

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology...... accurately and efficiently

Consequences

Specific wording in questions to **prevent** the use of calculators

(ii) Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} \sin 2x + \sec\frac{1}{3}x\tan\frac{1}{3}x\,dx$$

(b) Use calculus to find the coordinates of A.

(ii) Find $\int_1^2 f(x)\,dx$, giving your answer in the form $a + \ln b$, where a and b are constants.

Find, by integration, the exact value for the area of R.

Give your answer in terms of $\ln 2$

Looking at AO5 on P1, P2, P3 and P4

Use contemporary calculator technology…… accurately and efficiently

Consequences:

Specific wording in questions to **prevent** the use of calculators

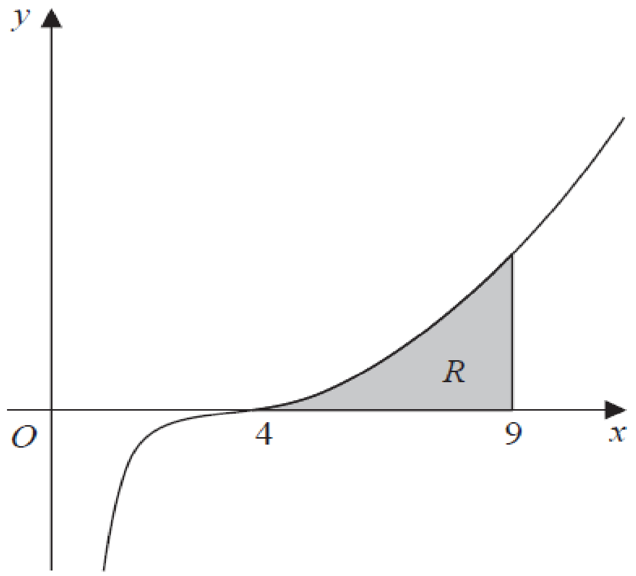


Figure 1

In this question you must show all steps of your working.

Solutions relying on calculator technology are not acceptable.

Looking at AO5 on P1, P2, P3 and P4

…and other permitted resources (such as formulae booklets…)

From the P1 specification

One form of the Cosine Rule

Students have to learn the quadratic formula and the sine rule (and possibly the alternative form of the cosine rule)

Looking at AO5 on P1, P2, P3 and P4

…and other permitted resources (such as formulae booklets…)

From the P2 specification

nth terms and sums of arithmetic and geometric series

Change of base rule for logs

Binomial series (both forms)

Trapezium rule.

Students have to learn

Another three laws of logs*
Two trig formulae
Integral form of area under a curve.

Looking at AO5 on P1, P2, P3 and P4

…and other permitted resources (such as formulae booklets…)

From the P3 specification

Trig identities

e.g.. sin (A + B) = sin A cos B + sin B cos A

Students have to learn

Several trig identities

Looking at AO5 on P1, P2, P3 and P4

…and other permitted resources (such as formulae booklets…)

From the P4 specification

Scalar product of 2 column vectors

Students have to learn

Mark Schemes

Mark schemes

M marks – are awarded for a correct method or an attempt at a correct method

A marks – are dependent accuracy marks. They are dependent on the **M** mark being awarded.

The marking pattern **M0A1 is IMPOSSIBLE**

B marks – are independent accuracy marks [or sometimes just the answer] . For example, for a comment or for a graph.

A and B marks may be follow through [ft]

In addition: bod means benefit of doubt, **ft** means follow through, **cao** means correct answer only, **cso** means correct solution only, **isw** means ignore subsequent working [**provided it does not contradict an answer already given**], **awrt** means answers which round to.

Marking activity

Activity 4

Mark these three examples using the **mark scheme only**.

On the scripts, mark the place at which you award the marks.

We will go through the marks at the end of the activity.

Marking activity

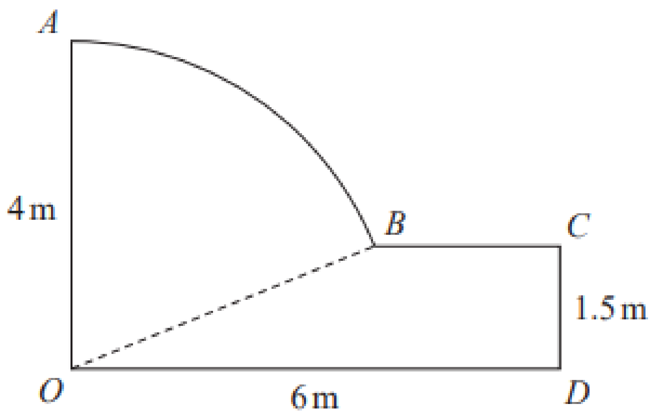


Figure 1

Figure 1 shows the plan for a garden.

In the plan

- OA and CD are perpendicular to OD
- AB is an arc of the circle with centre O and radius 4 metres
- BC is parallel to OD
- OD is 6 metres, OA is 4 metres and CD is 1.5 metres

(a) Show that angle AOB is 1.186 radians to 4 significant figures.

(2)

(b) Find the perimeter of the garden, giving your answer in metres to 3 significant figures.

(4)

(c) Find the area of the garden, giving your answer in square metres to 3 significant figures.

(4)

Marking activity Script 1

(a)

M1: Complete correct method to find angle AOB

A1*: Correct work to obtain 1.186

(b)

M1: For 1.186 x 4

M0: No correct work seen for BC. (It is not clear where the 4.630 comes from)

ddM0A0: Follows

(c)

M1: Uses the correct sector area formula

M1: Correct method

for the area of OBCD using a trapezium using their BC which was found using trigonometry

ddM1: Correct method for the total area.

A0: Incorrect

Marking activity Script 2

(a)

M1: Correct method for angle AOB

A1*: Correct work to obtain 1.186

(b)

M1: For

1.186 x 4

M1: Correct method for BC (6 - root(4^2 - 1.5^2))

ddM1: Full correct method for

the perimeter

A1: For awrt 18.5

(c)

M0: Incorrect method for the sector area. They use

1.256 not 1.186

M1: Correct method for the area of OBCD using a rectangle + triangle following an

attempt at BC using Pythagoras.

ddM0A0: Follows first M0

Marking activity Script 3

(a)

M1: Correct method for angle AOB

A1*: Obtains 1.186 correctly.

(b)

M1: For 4 x

1.186

M0: BC is never found

ddM0A0: Follows

(c)

M1: Uses the correct sector area

formula

M0: They do not have a value for BC so this mark is not available

ddM0A0: Follows

Resources and Support

Results Plus

- Provides detailed analysis of your learners performance.
- Identify potential topics, skills and types of question where students may need to develop their learning further.
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- Understand how your students' performance compares with class and Pearson Edexcel national averages.
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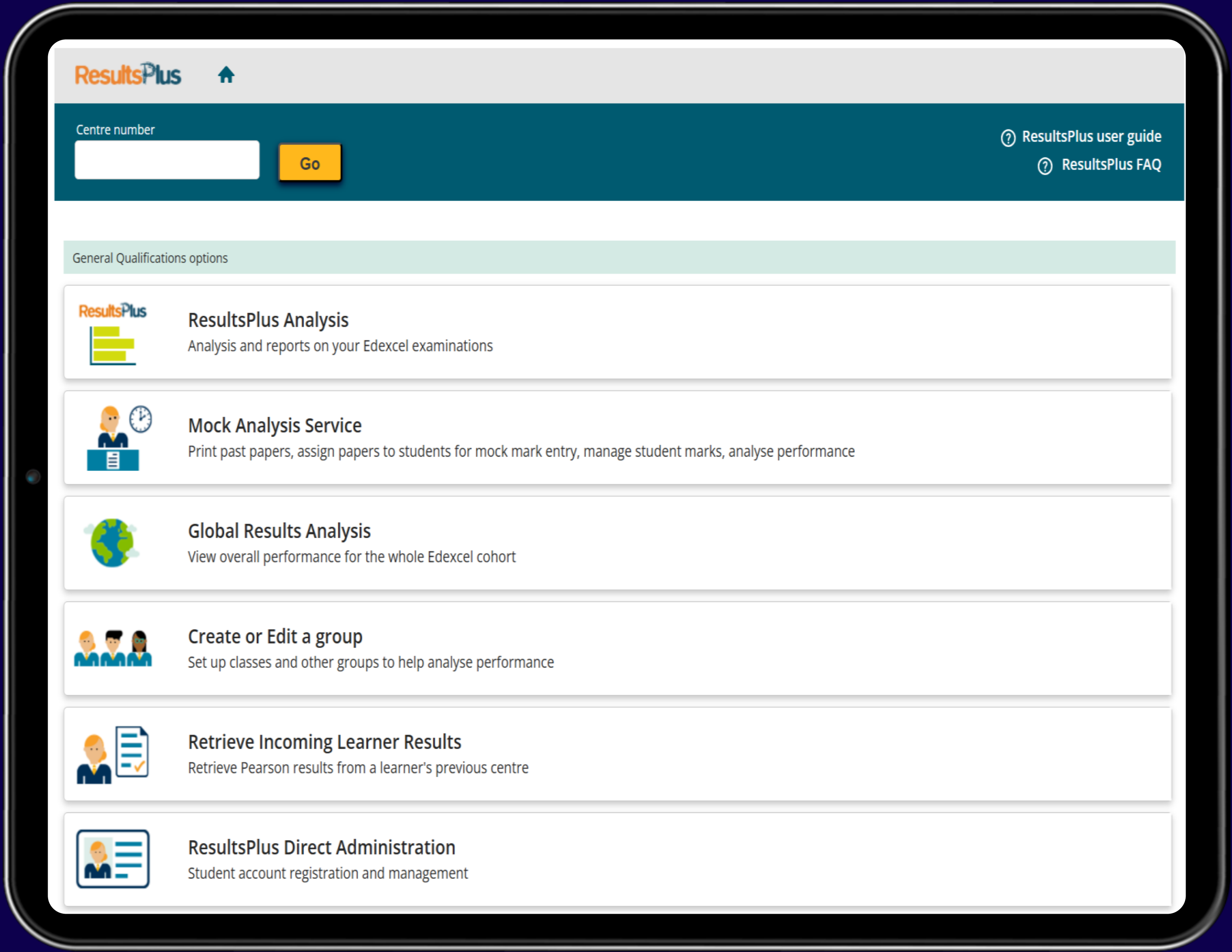
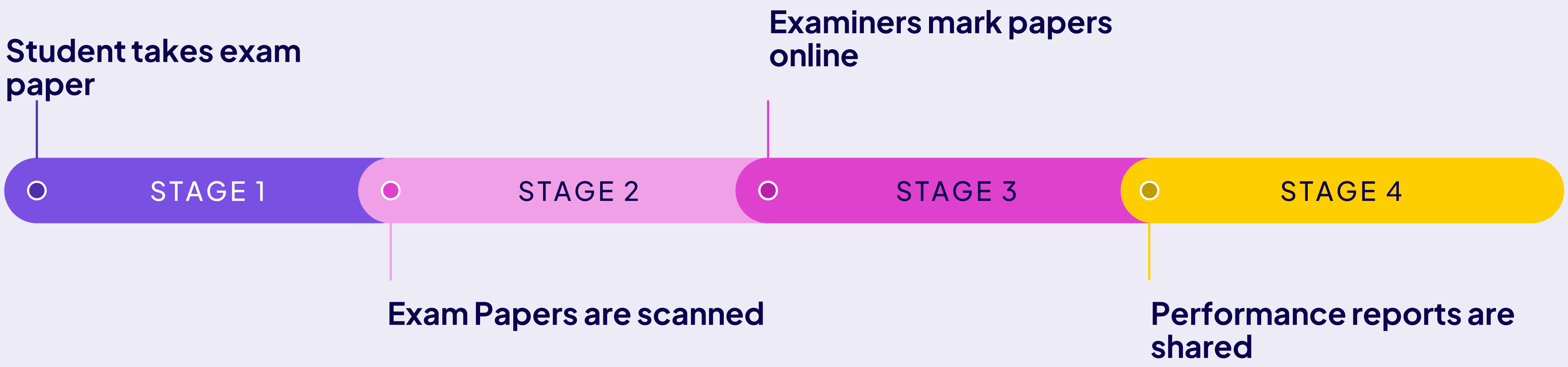


Results Plus



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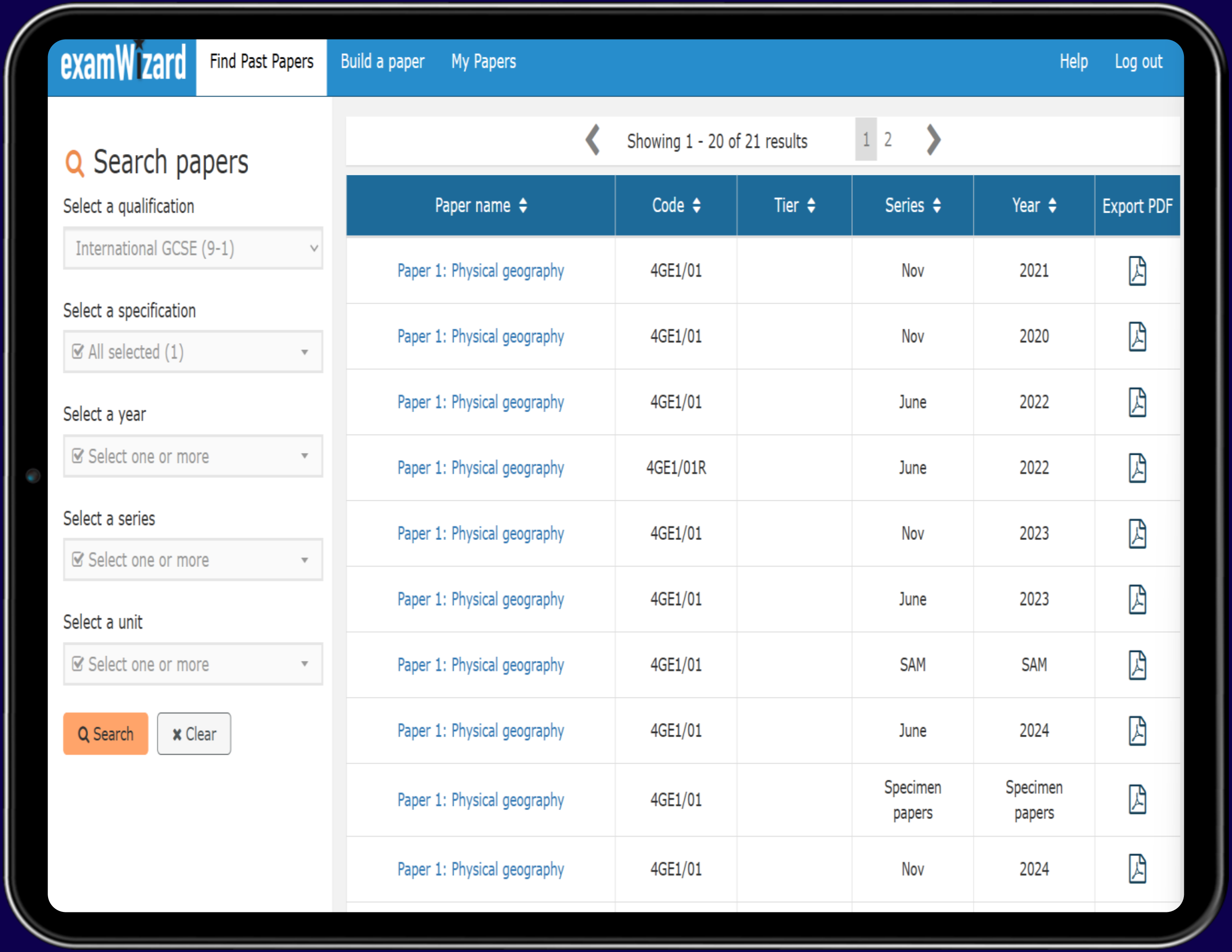

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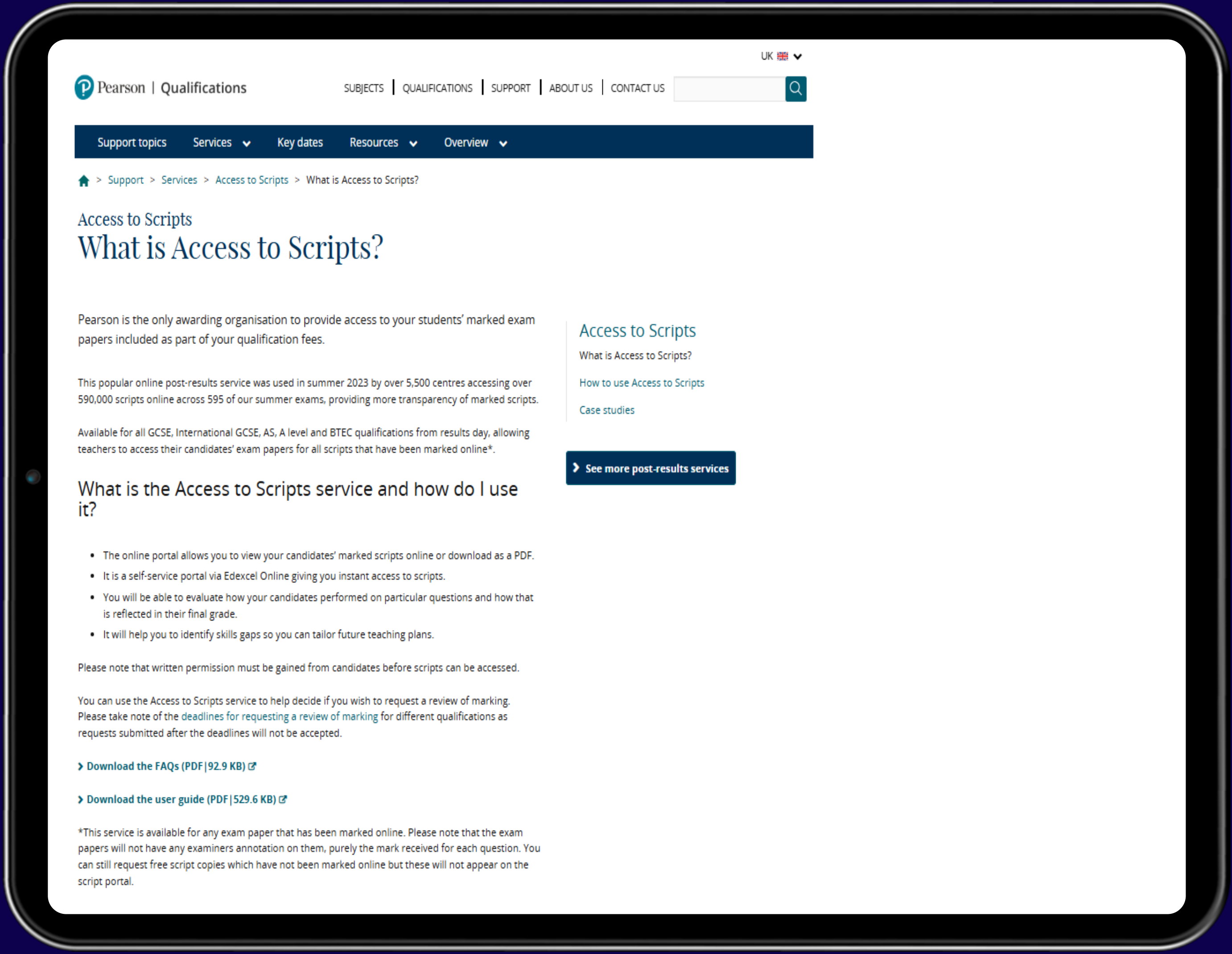

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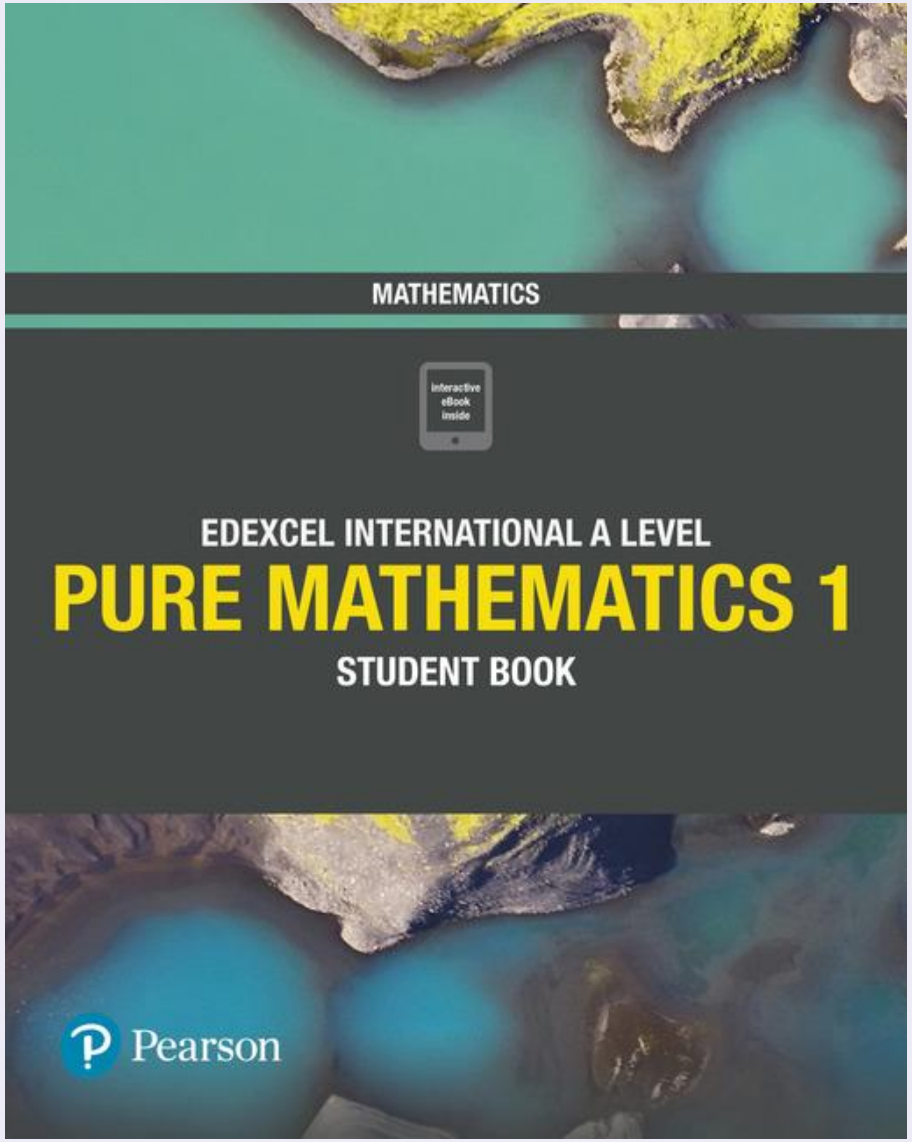

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